PROSPECTS FOR IMPROVED Λ_c BRANCHING FRACTIONS

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The experimental uncertainty on the branching fraction $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$ has not decreased since 1998, despite a much larger data sample. Uncertainty in this quantity dominates that in many other quantities, including branching fractions of Λ_c to other modes, branching fractions of b-flavored baryons, and fragmentation fractions of charmed and bottom quarks. Here we advocate a lattice QCD calculation of the form factors in $\Lambda_c \to \Lambda\ell^+\nu_\ell$ (the case $\ell = e^+$ is simpler as the mass of the lepton can be neglected). Such a calculation would yield an absolute prediction for the rate for $\Lambda_c \to \Lambda\ell^+\nu_\ell$. When combined with the Λ_c lifetime, it could provide a calibration for an improved set of Λ_c branching fractions as long as the accuracy exceeds about 25%.

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I INTRODUCTION

Despite the accumulation of a vastly greater sample of charmed particles in e^+e^-, ep, and hadron-hadron collisions, the most accurately known branching fraction for the decay of the lowest-lying charmed baryon Λ_c, $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$, has remained at the same value since 1998. It was only pinned down to that accuracy thanks to constructive suggestions by Dunietz [1]. This branching fraction sets the scale for a number of other quantities which depend on it. Many other Λ_c branching fractions are measured through their ratio to the $pK^-\pi^+$ mode [2]. It sets the scale for b-flavored baryon branching fractions, and governs fragmentation fractions of charm and bottom quarks into baryons.

In the present paper we advocate improvement of accuracy of the semileptonic branching fraction $\mathcal{B}(\Lambda_c \to \Lambda e^+\nu_e)$, whose current value is $(2.1 \pm 0.6)\%$, via a lattice QCD calculation of the relevant form factors. Such calculations have been performed for the semileptonic decays of charmed mesons, $D \to K\ell\nu_\ell$ and $D \to \pi\ell\nu_\ell$ [3], which are characterized by two form factors. Although four form factors are relevant to $\Lambda_c \to \Lambda\ell\nu_\ell$ in the limit of zero lepton mass, the difficulty of such a calculation is outweighed by its importance. A calculation enabling the prediction of the rate for $\Lambda_c \to \Lambda e^+\nu_e$ (and hence its branching fraction, given $\tau(\Lambda_c) = 200 \pm 6$ fs [2]) to an accuracy of better than about 25% would represent an improvement on a wide variety of key quantities.

In Section II we review various quantities which could profit from improvement in the accuracy of $\mathcal{B}(\Lambda_c \to \Lambda e^+\nu_e)$. We discuss in Section III the present status of understanding of form factors in this decay. The corresponding semileptonic decay $\Lambda_b \to \Lambda_c e^-\bar{\nu}_e$, to which the Heavy Quark Effective Theory (HQET) can be applied, is treated in Section IV. Some remarks are made in Section V regarding the "calibrating" mode $\Lambda_c \to pK^-\pi^+$, while Section V concludes.

II DEPENDENT QUANTITIES

A Λ_c branching fractions

Many Λ_c branching fractions are determined by their ratio with respect to $\mathcal{B}(\Lambda_c \to pK^-\pi^+)$ [2]: For example,

$$\frac{\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = 0.41 \pm 0.07 \; ; \quad \frac{\mathcal{B}(\Lambda_c \to \Lambda \pi^+)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = 0.204 \pm 0.019 \; . \tag{1}$$

In the last quantity we use the Particle Data group "average." As $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$ is known to only a fractional error of 26%, this limits the accuracy to which quantities depending on it can be determined. Other ratios [2] are

$$\frac{\mathcal{B}(\Lambda_c \to \Lambda \pi^+ \pi^+ \pi^-)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = 0.522 \pm 0.032 \; ; \quad \frac{\mathcal{B}(\Lambda_c \to p\bar{K}^0)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)} = 0.47 \pm 0.04 \; , \tag{2}$$

using "average" values in both cases. We advocate instead making a modest improvement in the first ratio of Eq. (1) and calibrating Λ_c branching fractions by the $\Lambda e^+ \nu_e$ mode.

B Λ_b branching fractions

Most tabulated Λ_b branching fractions involve a Λ_c in the final state [2]. (An exception is the recently observed decay $\Lambda_b \to \Lambda \mu^+ \mu^-$ [4].) Examples are

$$\mathcal{B}(\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}_\ell) = 0.050^{+0.011+0.016}_{-0.008-0.012}$$

[5],

$$\mathcal{B}(\Lambda_b \to \Lambda_c^+ \pi^-) = (8.8 \pm 2.8 \pm 1.5) \times 10^{-3} \tag{3}$$

[6]. The former measurement makes use of the branching fractions of Λ_c to $pK^-\pi^+$, $\Lambda \pi^+ \pi^+ \pi^-$, and pK_S, while the latter employs only the $pK^-\pi^+$ mode. As $\mathcal{B}(\Lambda \pi^+ \pi^+ \pi^-)$ and $\mathcal{B}(pK_S)$ are quoted with respect to $\mathcal{B}(pK^-\pi^+)$, their accuracies are limited as well.

C Fragmentation fractions

Individual probabilities for $c \to (D^0, D^+, D_s^+, \Lambda_c, \ldots)$ do not seem to have been quoted in the literature. However, the corresponding fractions for $b \to (\overline{B}^0, B^-, \overline{B}^0_s, \Lambda_b)$ are noted in the Particle Data Group's mini-review on B^0–$\bar{B}^0$ mixing [1] and in a recent study by the LHCb Collaboration [7]. Such fractions are needed in a wide variety of applications, including the interpretation of CP asymmetries in same-sign dimuon production at the Tevatron [8], and in studies of top quark production.

As an illustration of the uncertainty associated with Λ_c branching fractions, Ref. [7] finds the ratio of strange B meson to light B meson $(\overline{B}^0, B^-)$ production to be

$$\frac{f_s}{f_u + f_d} = 0.134 \pm 0.004^{+0.011}_{-0.010} \; , \tag{4}$$

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[1]In Ref. [2] see the mini-review by Schneider, pp. 973–980, Table 1.

but a much larger error in the ratio of Λ_b to light meson production:

$$\frac{f_{\Lambda_b}}{f_u + f_d} = [0.404 \pm 0.017(\text{stat}) \pm 0.027(\text{syst}) \pm 0.105(\text{Br})]$$

$$\times [1 - (0.031 \pm 0.004(\text{stat}) \pm 0.003(\text{syst}))p_T(\text{GeV})] . \tag{5}$$

The uncertainty labeled "Br" is due to the 26% uncertainty in the branching fraction of Λ_c to $pK^-\pi^+$. An additional theoretical uncertainty is associated with the assumption that the total semileptonic widths of Λ_b and light B are equal up to a small correction ξ. Denoting a generic charmed hadron by D, Ref. [7] finds

$$\frac{f_{\Lambda_b}}{f_u + f_d} = \frac{n_{\text{corr}}(\Lambda_b \to D\mu)}{n_{\text{corr}}(B \to \overline{D}^0\mu) + n_{\text{corr}}(B \to D^-\mu)} \frac{\tau_{B^-} + \tau_{B^-}}{2\tau_{\Lambda_b}}(1 - \xi) , \tag{6}$$

quoting $\xi = (4 \pm 2)\%$. Examples of results for ξ using heavy-quark and operator product expansions are $(2.1 \pm 0.6)\%$ [9], $(5.2 \pm 0.6)\%$ [10], and $\simeq 3\%$ [11]. A simple kinematic model, by contrast, gives $\xi \simeq 11\%$ [12]. This, parenthetically, emphasizes the importance of measurement of the inclusive branching fraction $\mathcal{B}(\Lambda_b \to \ell^-\bar{\nu}_\ell X)$, for which a value has never been quoted. The inclusive branching fraction $\mathcal{B}(\Lambda_c \to \ell^+\nu_\ell X)$ is not particularly well known either [2, 12]:

$$\frac{\Gamma(\Lambda_c \to e^+\nu_e X)}{\overline{\Gamma}(D \to e^+\nu_e X)} = 1.44 \pm 0.54 \tag{7}$$

[$\overline{\Gamma}$ denotes a (D^0, D^+) average]. This ratio is to be compared with the prediction of 1.67 in the model of Ref. [12] and about 1.2 based on a heavy-quark expansion including $1/m_c^2$ terms [9]. It would be highly worthwhile to improve the precision of these measurements, an effort well within the capabilities of the BaBar and Belle Collaborations.

III FORM FACTORS IN $\Lambda_c \to \Lambda e^+\nu_e$

For a semileptonic decay of one spin-1/2 hadron to another there are three vector and three axial-vector form factors. One of each is negligible in the limit of zero lepton mass, which we shall assume. There remain two vector and two axial-vector form factors, but for an arbitrary semileptonic decay $\Lambda_1 \to \Lambda_2\ell\nu_\ell$ in the heavy-quark limit of Λ_1 all form factors appear multiplying a factor $1 - \gamma_5$ and hence the vector and axial-vector form factors are equal pairwise. The weak current matrix element then may be written [13] as

$$\langle\Lambda_2|J_\mu^{V+A}|\Lambda_1\rangle = \bar{u}(P_2)[f_1(q^2)\gamma_\mu(1 - \gamma_5) + f_2(q^2)\slashed{v}_1(1 - \gamma_5)]u(P_1) , \tag{8}$$

We have denoted the (initial,final) Λ by $\Lambda_{(1,2)}$ with four-momentum $P_{(1,2)}$, mass $M_{(1,2)}$, and covariant four-velocity $v_{(1,2)} = P_{(1,2)}/M_{(1,2)}$. The four-momentum transfer to the lepton pair is $q = P_1 - P_2$. (In the heavy-quark limit for the final Λ, $f_2 = 0$ and $f_1 = 1$ at $q^2 = q^2_{\max}$.) The form factors are assumed to be in a constant ratio $r = f_2/f_1$, and to be governed by a dipole structure in q^2. With the choice of the D_s^* mass in the dipole form factor, the rate for $\Lambda_c \to \Lambda e^+\nu_e$ is then predicted to be

$$\Gamma(\Lambda_c \to \Lambda e^+\nu_e) = \begin{cases} 1.57 \times 10^{11} \text{ s}^{-1} & \text{for } r = 0 , \\ 1.90 \times 10^{11} \text{ s}^{-1} & \text{for } r = -0.25 , \end{cases} \tag{9}$$

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where the latter value is preferred on the basis of an expansion in the inverse of the strange quark mass (admittedly a crude approximation).

Experimental information on the decay $\Lambda_c \to \Lambda e^+\nu_e$ comes from the ARGUS [14] and CLEO [15] Collaborations:

$$\frac{\Gamma(\Lambda_c \to \Lambda e^+\nu_e)}{\Gamma(\Lambda_c \to pK^-\pi^+)} = \begin{cases} 0.38 \pm 0.14 \ [14] \\ 0.42 \pm 0.07 \ [15] \\ 0.41 \pm 0.07 \ [2] \end{cases} . \tag{10}$$

Combining the last of these (the PDG average) with $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$ [2] and the Λ_c lifetime $\tau(\Lambda_c) = (200 \pm 6)$ fs [2] one finds the experimental value

$$\Gamma(\Lambda_c \to \Lambda e^+\nu_e) = (1.03 \pm 0.32) \times 10^{11} \ \text{s}^{-1} \ , \tag{11}$$

somewhat below the predictions of Ref. [13].

To give a qualitative idea of the expected shape of the leading form factor (the one which does not vanish in the limit of heavy initial and final quarks in the Λ), we adapt a discussion of the decay $\Lambda_b \to \Lambda_c e^-\bar{\nu}_e$ [16] to the case of $\Lambda_c \to \Lambda e^+\nu_e$, treating the strange quark in the Λ as "heavy". (As shown, for example, in Ref. [17] for charmed meson semileptonic decays, this assumption has limited validity.)

The Isgur-Wise [18] variable $w = v_1 \cdot v_2$ is related to q^2 by

$$w = \frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2} \tag{12}$$

and is equal to 1 at the zero-recoil point $q^2 = Q^2_{\max} = (M_1 - M_2)^2$.

The differential decay rate in the heavy-quark limit and the limit of vanishing final lepton mass is [16]

$$\frac{d\Gamma(\Lambda_1 \to \Lambda_2 \ell\nu_\ell)}{dw} = \frac{G_F^2 M_1^5 |V_{ij}|^2}{12\pi^3} r^3 \sqrt{w^2 - 1}[3w(1 + r^2) - 2r(1 + 2w^2)] \, \zeta(w)^2 \ , \tag{13}$$

where V_{ij} is the appropriate CKM matrix element for the semileptonic quark transition $i \to j\ell\nu_\ell$, $r \equiv M_2/M_1$, and $\zeta(w)$ is the Isgur-Wise function, normalized to $\zeta(1) = 1$. A simple form which we shall adopt is $\zeta(w) = \exp[-\rho^2(w-1)]$. Taking [2] $M_1 = 2.28646$ GeV, $M_2 = 1.115683$ GeV, $|V_{cs}| = 0.97343$, we find the central value of Eq. (11) is reproduced for $\rho^2 = 4.75$. The corresponding spectrum for $d\Gamma/dw$ is shown in Fig. 1. A similar shape is to be expected for a realistic lattice gauge theory calculation, which should take into account the contributions of form factors which vanish in the heavy-quark limit.

The lattice calculation of form factors in $\Lambda_c \to \Lambda\ell\nu_\ell$ may prove to be quite challenging. For $D \to K\ell\nu_l$, errors in form factors of several percent have been achieved [3]. One could hope for the baryonic case to be similar with the replacement of a light antiquark spectator in $D \to K$ by a ud diquark with $I = J = 0$ in $\Lambda_c \to \Lambda$. However, the ud diquark can undergo internal excitations, making the situation more complicated than in the mesonic case. A note of caution is also provided by the current status of the lattice calculation of semileptonic Λ_b decays, which we now discuss briefly.

IV   FORM FACTORS IN $\Lambda_b \to \Lambda_c e^-\bar{\nu}_e$

The calculation of the previous section can be adapted to the decay $\Lambda_b \to \Lambda_c \ell\nu_\ell$, for which the heavy-quark limit should be a better approximation. We take [2] $M_1 = 5.6202$

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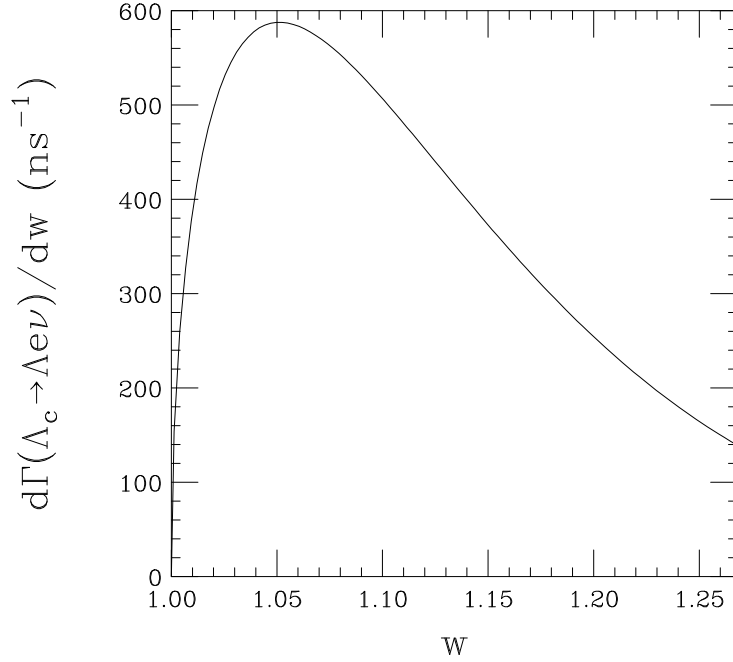Figure 1: Differential decay rate for $\Lambda_c \to \Lambda \ell \nu_\ell$ with respect to Isgur-Wise variable w, with the Isgur-Wise function $\zeta(w) = \exp[-4.75(w-1)]$ reproducing the central value of the observed decay rate (11).

GeV, $M_2 = 2.28646$ GeV, and $|V_{cb}| = 0.041$. The experimental branching fraction is $\mathcal{B}(\Lambda_b \to \Lambda_c e^- \bar{\nu}_e) = (5.0^{+1.9}_{-1.4})\%$; combined with the Λ_b lifetime $\tau(\Lambda_b) = (1.425 \pm 0.032) \times 10^{-12}$ s, this gives a decay rate

$$\Gamma(\Lambda_b \to \Lambda_c e^- \bar{\nu}_e) = (3.5^{+1.3}_{-1.0}) \times 10^{10} \text{ s}^{-1} \ , \tag{14}$$

whose central value is reproduced with the choice $\rho^2 = 2.3$ in the Isgur-Wise function. A similar though not identical result is obtained by the DELPHI Collaboration [5]. The corresponding differential decay rate is shown in Fig. 2.

There exists a lattice QCD study of the decay $\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$ [19]. The function $\zeta(w)$, if normalized to 1 at $w = 1$, is seen to fall to 0.65 ± 0.03 at $w = 1.06$, corresponding to $\rho^2 = 7.2 \pm 0.8$. This is quite far from the value which reproduces the observed decay rate. It is not clear whether this is an intrinsic shortcoming of the lattice approach, which would bode poorly for calculating $\Gamma(\Lambda_c \to \Lambda e^+ \nu_e)$ to better than 25%, or a feature of the specific calculation which might be improved using more recent techniques.

V   REMARKS ON THE MODE $\Lambda_c \to pK^-\pi^+$

The decay $\Lambda_c \to \Lambda e^+ \nu_e$ has one disadvantage with respect to all-hadronic modes such as $\Lambda_c \to pK^-\pi^+$: In the semileptonic decay, one must ensure that nothing besides the neutrino is missing, whereas an all-charged mode such as $pK^-\pi^+$ provides a useful kinematic constraint. It is therefore worth reviewing briefly the ingredients in the present determination of the "calibrating" branching fraction $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$ [2], to see if
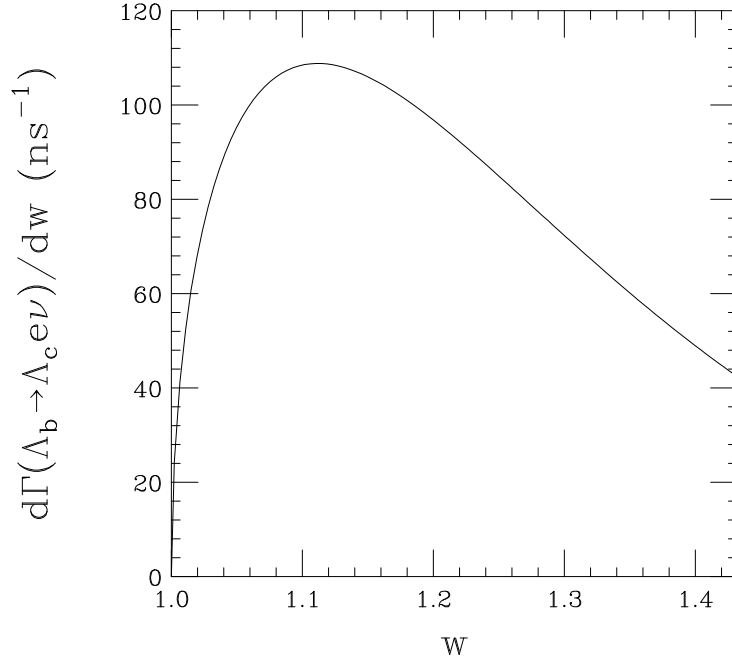
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[2]See Burchat, mini-review in Ref. [2], pp. 1260–1261.

Figure 2: Differential decay rate for $\Lambda_b \to \Lambda_c \ell \nu_\ell$ with respect to Isgur-Wise variable w, with the Isgur-Wise function $\zeta(w) = \exp[-2.3(w-1)]$ reproducing the central value of the observed decay rate (14).

some improvement in that quantity is possible.

One determination of $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 1.3)\%$ is obtained by averaging two types of measurements. The first measures a combined branching ratio $\mathcal{B}(\bar{B} \to \Lambda_c X) \cdot \mathcal{B}(\Lambda_c \to pK^-\pi^+)$ and estimates the first factor by assuming that $\Lambda_c X$ final states other than $\Lambda_c \overline{N} X$ are negligible. This assumption was called into question in Ref. [1]. The second relies upon measurement of the ratio (10) and the assumptions that (i) the semileptonic decay of Λ_c is saturated by the $\Lambda e^+ \nu_e$ final state, and (ii) all inclusive semileptonic decay rates of charmed particles are equal. While this appears to be true for mesons, it is far from established in the case of Λ_c [12].

An independent determination of $\mathcal{B}(\Lambda_c \to pK^-\pi^+) = (5.0 \pm 0.5 \pm 1.2)\%$ was performed by the CLEO Collaboration [20]. It analyzes $e^+e^- \to c\bar{c} \to \bar{D}\bar{p}X$ continuum events, where the $\bar{c}$ is tagged by the presence of the $\bar{D}$, the $\bar{p}$ is in the hemisphere opposite to the $\bar{D}$ (to reduce non-signal background), and it is assumed that there is always a Λ_c present in X to compensate for charm and baryon number. One then measures the $pK^-\pi^+$ yield in the same hemisphere as the $\bar{p}$ to obtain $\mathcal{B}(\Lambda_c \to pK^-\pi^+)$. Backgrounds against which one has to guard include $D\bar{D}N\bar{p}$ and kaons producing fake antiproton tags.

The measurement of Ref. [20] is based on 3.1 fb^{-1} collected at the $\Upsilon(4S)$ resonance and 1.6 fb^{-1} collected about 60 MeV below it, corresponding to about 5 million continuum $c\bar{c}$ events. Although the experimental error is dominated by systematics, the authors note that more data would allow better understanding of backgrounds such as $D\bar{D}N\bar{p}$. It would be worth seeing how well one could perform such an analysis with the much larger data samples available to the BaBar and Belle Collaborations.

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VI CONCLUSIONS

The importance of improved knowledge of the decay rate for $\Lambda_c \to \Lambda e^+ \nu_e$ has been stressed. Progress is possible in principle upon a variety of fronts, including (1) lattice gauge theory calculations of form factors, (2) improved measurements of ratios of Λ_c branching fractions, (3) improved determination of inclusive Λ_c and Λ_b semileptonic branching fractions, and (4) validation of lattice QCD calculations and heavy-quark symmetry through the continued study of $\Lambda_b \to \Lambda_c \ell \nu_\ell$. Many quantities depend upon an absolute calibration of Λ_c branching fractions, a goal whose attainment is long overdue.

ACKNOWLEDGMENTS

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